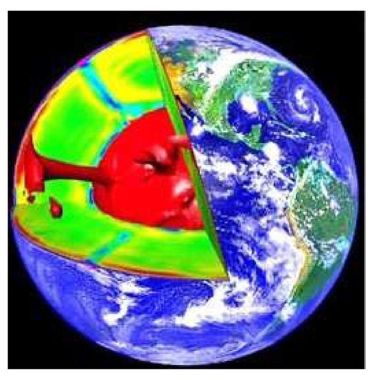

Simulation of Geophysical Problem at the Parallel Supercomputer

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Mathematical Statement of the Thermal Convection Problem

Governing Equations

Model domain $\Omega = (0, x_1 = l_1) \times (0, x_2 = l_2) \times (0, x_3 = h)$, $t \in (0, \vartheta)$. The boundary-value problem for flow velocity:

$$-\nabla P + \nabla\cdot(\eta(T)[\nabla \mathbf{u} + \nabla \mathbf{u}^T]) + (RaT - La\Phi(T))\mathbf{e_3} = \mathbf{0}, \quad (1)$$

$$\nabla\cdot\mathbf{u} = 0, \quad x\in\Omega; \quad Ra = \frac{\alpha g \rho_* T_* h^3}{\eta^* \kappa},$$

$$\mathbf{u}\cdot\mathbf{n} = 0, \quad \partial\mathbf{u}_\tau/\partial\mathbf{n} = 0, \quad x\in\partial\Omega; \quad La = \frac{Ra}{\alpha\Delta T}. \quad (2)$$

Phase changes at 410km and 660 km boundaries

$$\rho(T,\mathbf{x}) = \rho_*(1 - \alpha T + \Phi(T)), \quad \Phi(T) = \sum_{i=1,2} a_i \Phi_i(T),$$

$$\Phi(\pi) = 1/2\left[1 + \tanh\frac{\pi_i}{w_1}\right], \quad \pi_i = z_i - x_3 - \gamma_i(T - T_i), \quad i = 1,2,$$

where ρ_* is the reference density; α is the coefficient of thermal expansion; $a_1 = 0.05$; $a_2 = 0.09$; $h = 800$ km; $z_1 = 410km$; $z_2 = 660$ km; $w_1 = w_2 = 10$ km; $\gamma_1 = -4MPaK^{-1}$ and $\gamma_2 = 2MPaK^{-1}$; $T_1 = 1790$K; $T_2 = 1858$K; g is the acceleration due to gravity.
The initial-boundary-value problem for temperature:

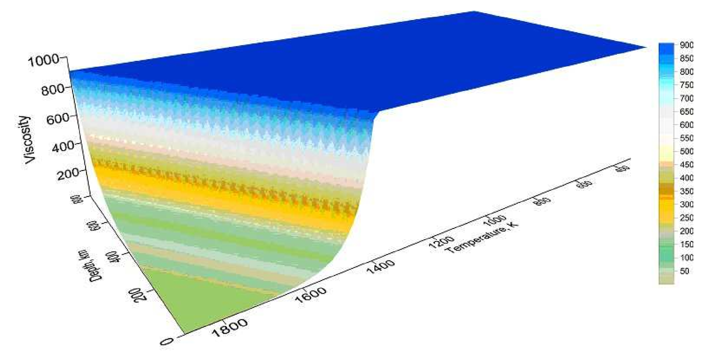
$$\frac{\partial T}{\partial t} + \mathbf{u}\cdot\nabla T + A^{-1}BDi^*Rau_3T = A^{-1}\left(-\nabla^2 T + Di^*\eta\sum_{i,j=1}^{3}(e_{ij})^2\right), \quad (3)$$

$$A = \left[1 + \left(a_1\frac{d\Phi_1}{d\pi_1}\gamma_1^2 + a_2\frac{d\Phi_2}{d\pi_2}\gamma_2^2\right)Di^*LaT\right] > 0,$$

$$B = \left[1 + \frac{La}{Ra}\left(a_1\frac{d\Phi_1}{d\pi_1}\gamma_1^2 + a_2\frac{d\Phi_2}{d\pi_2}\gamma_2^2\right)\right],$$

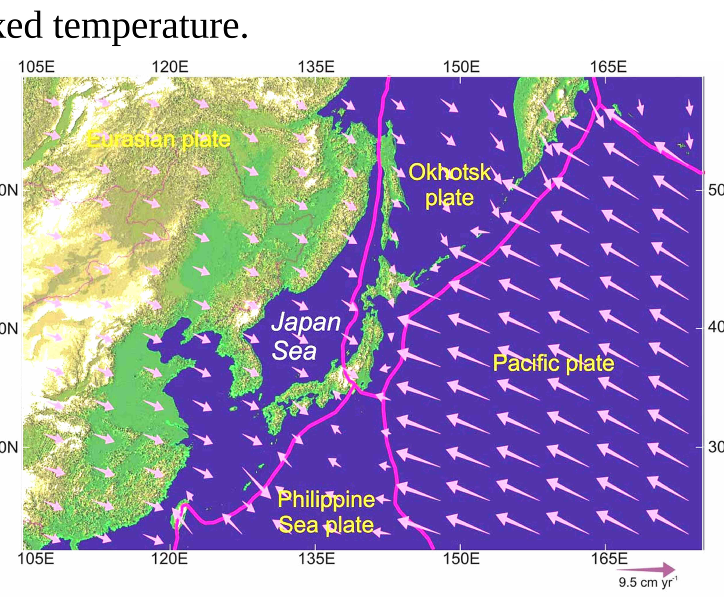
$$\sigma_1 T + \sigma_2 \partial T/\partial\mathbf{n} = T_*(t,\mathbf{x}), \quad T(0,\mathbf{x}) = T_0(\mathbf{x}).$$

Viscosity law $\eta(T,z) = \exp\left(\frac{E_a + V_a\rho g z}{RT}\right)$.



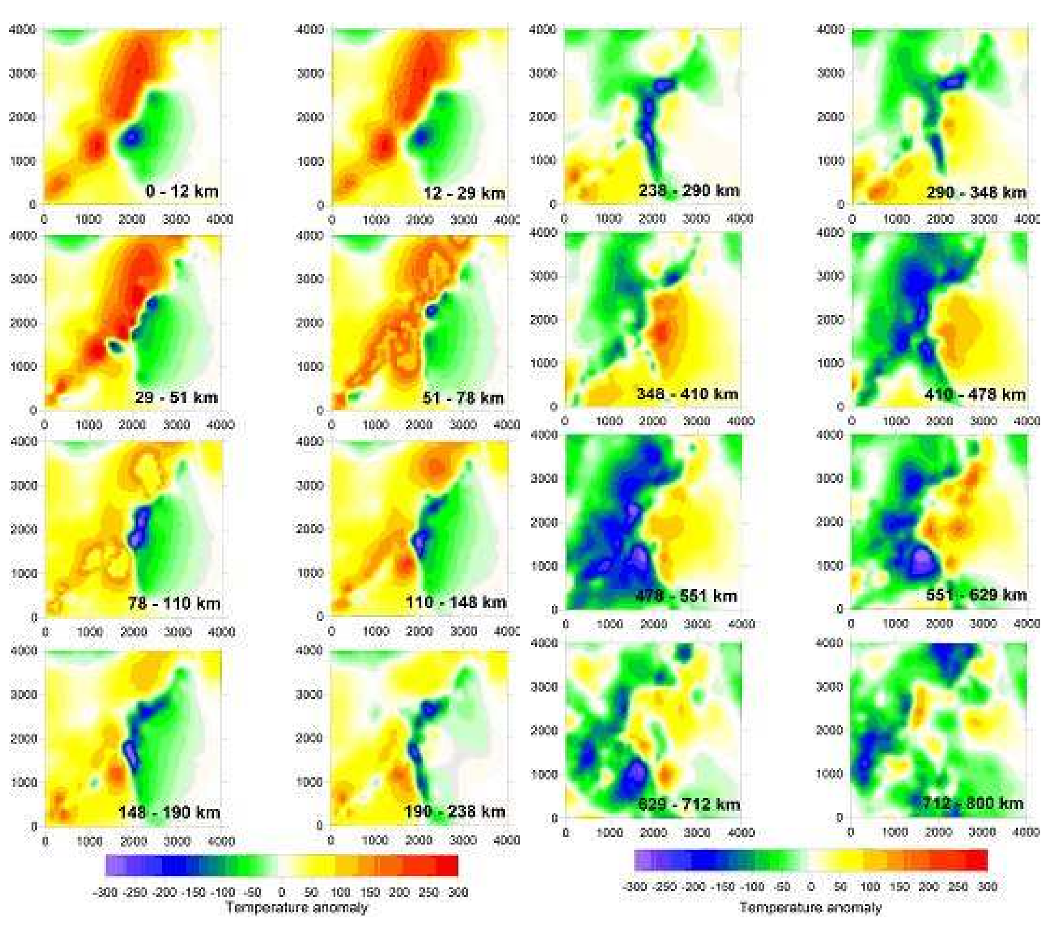
Boundary and Initial Conditions

Conditions at the top surface of the model boundary are prescribed velocity and fixed temperature.



Topography map of the Japanese Islands and surroundings. The plate motions and deformations are presented by arrows. The rate of the motions is determined from geodetic data and for the Philippine Sea plate from the PB2002 model. The white star indicates the place of sampling for geochemical analysis.

Conditions at the lower surface of the model boundary are no-slip and fixed temperature. Conditions at all side boundaries: $\frac{\partial\mathbf{u}}{\partial\mathbf{n}}$, $\frac{\partial T}{\partial\mathbf{n}}$ and $\frac{\partial P}{\partial\mathbf{n}}$.



The present temperature model beneath the Japanese Islands is developed by using the high-resolution seismic tomography (P-wave velocity anomalies) for the region.

Inverse Problem of Mantle Convection

- to restore the temperature evolution in the mantle;
- to reconstruct the history of movements of continental plates;
- to find a density distribution of the mantle in the geological past and to compare the observed and modelled amounts and rates of "true polar wander".

Numerical method and Solvers

The governing equations with the prescribed boundary and initial conditions are solved numerically by the finite-volume method using open source computational fluid dynamics software package OpenFoam (http://www.openfoam.com/). We use 200x200x190 finite volumes (rectangular hexahedrons), and hence a horizontal resolution of the model is 20 km x 20 km. The model domain is divided into five horizontal layers: Layer 1 (from the surface to the depth of 400 km), layer 2 (400-420 km), layer 3 (420-650 km), layer 4 (650-670 km), and layer 5 (670 to 800 km). Within each layer 60, 40, 35, 40, and 15 grid points are used. Therefore, a vertical resolution of the model varies from 0.5 to 8.67 km. The accuracy of the numerical solutions has been verified by several tests including grid changes, volume preservation, and principle of the maximum.

Several approaches have been developed to reconstruct the past thermal state and flow in the crust and mantle: backward adveion method, sequential filtering method, variational/adjoint method, and quasi-reversibility (QRV) method. Among these methods, the QRV method is less susceptible to a noise (small temperature perturbations) in restoration models. The QRV method for data assimilation introduces a new term in the heat balance equation (the first term in Eq. 3) to regularize the equation when solving it backward in time. The additional term describes heat flux relaxation.

Velocity $\mathbf{u}$ and pressure P are found from the equations (1) and (2) using the SIMPLE method. The regularized heat balance equation (3) is approximated by the Euler method using the implicit approximation of the advective term and the explicit approximation of the conductive term:

$$(\mathbf{E} + \beta\mathbf{D})^2\frac{T^{n+1} - T^n}{dt} + \mathbf{C}T^{n+1} - \mathbf{D}T^n + f(\mathbf{w}, T^n) = 0,$$

where the discrete operators $\mathbf{C} = -\mathbf{C}^*$ and $\mathbf{D} = \mathbf{D}^*$ approximate the advective and conductive terms, respectively. To solve the numerical scheme we use the splitting method introducing the convection/antidiffusion and regularization. The system of the discrete equations is solved by the Bi-Conjugate Gradient method using the incomplete LU-factorization as a pre-conditioner.

Computations

The numerical simulation was performed at supercomputer "Uran" (Institute of Mathematics and Mechanics UB RAS, Yekaterinburg, Russia). The open source computational fluid dynamics software package OpenFoam 2.0.1 was applied for numerical computations. Parallelisation is robust at a low level, and codes were run in parallel by default. For visualisation of OpenFOAM simulations, we develop a module for OpenFOAM data for the open source visualization application ParaView.

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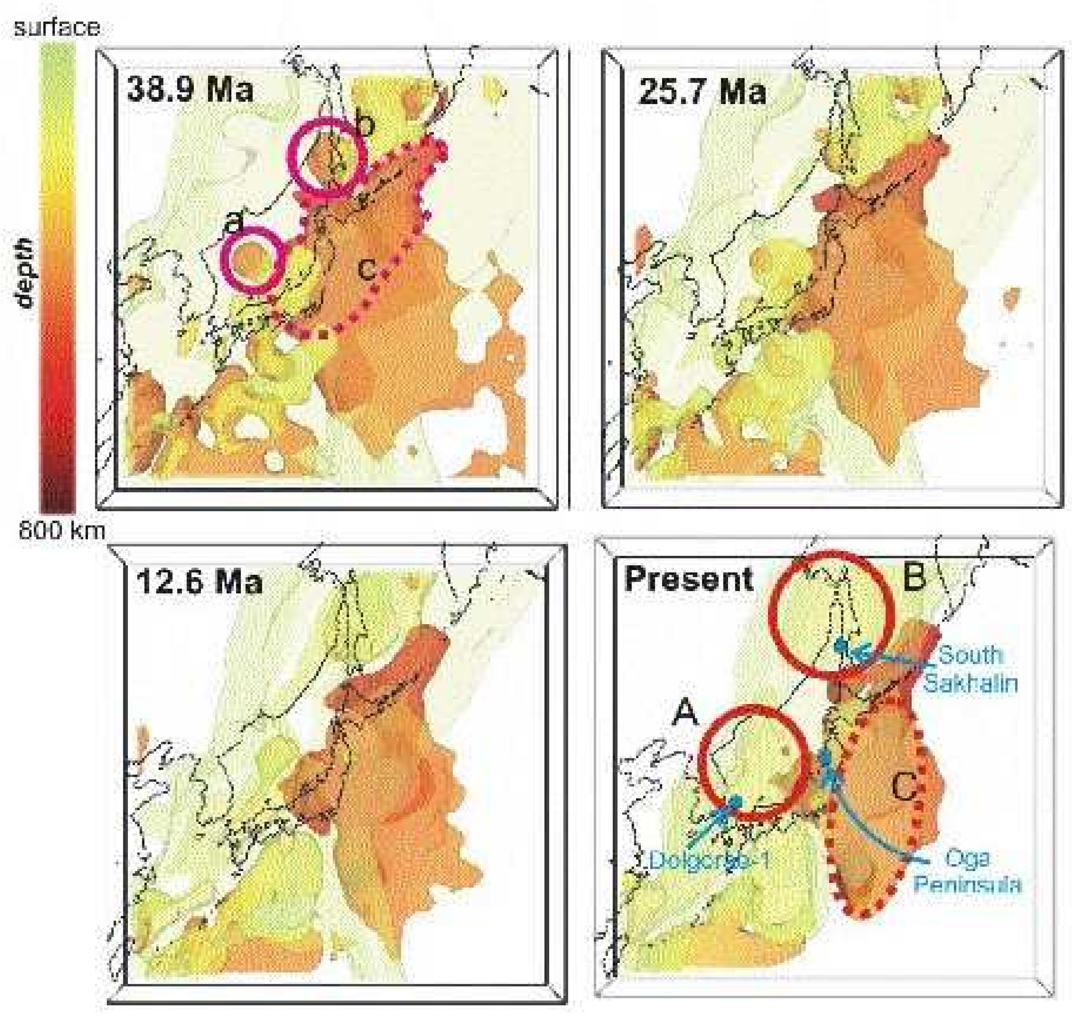
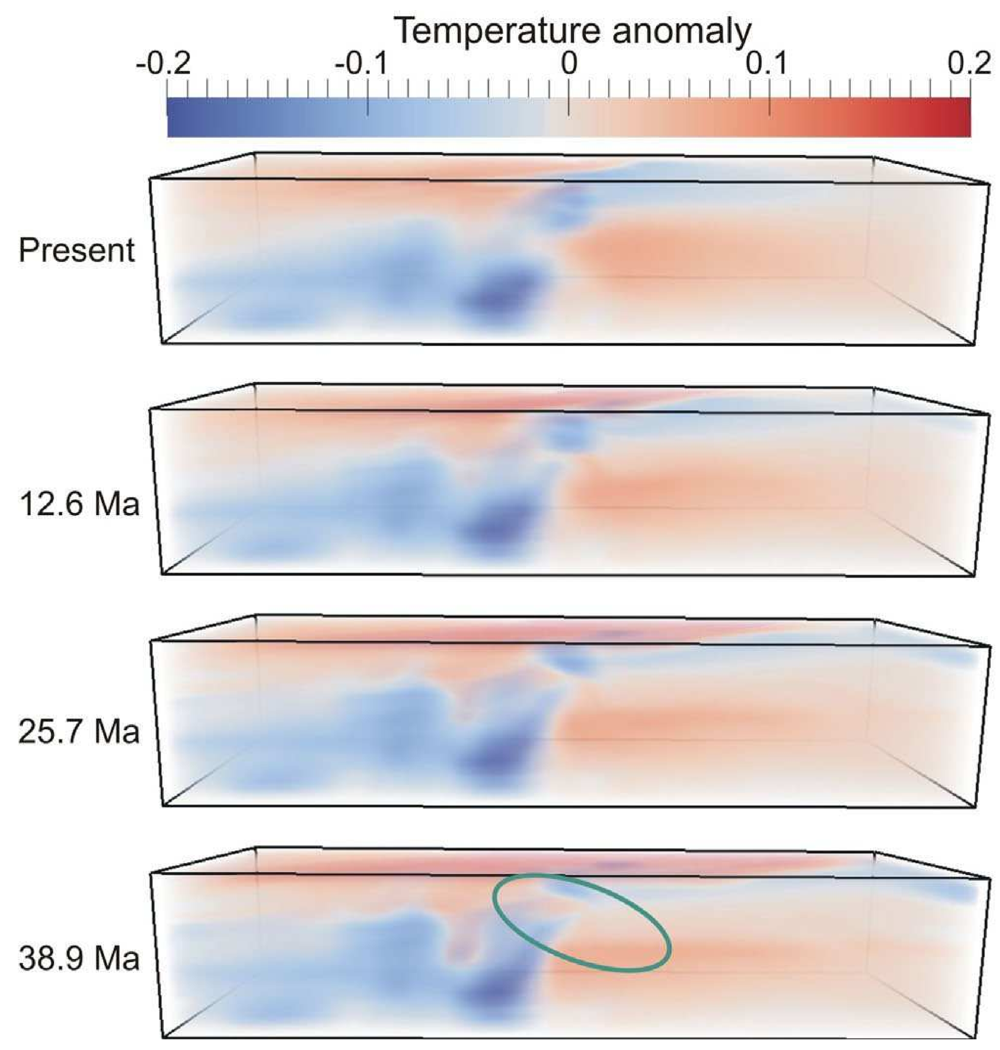
Results, visualization & scientific interpretation

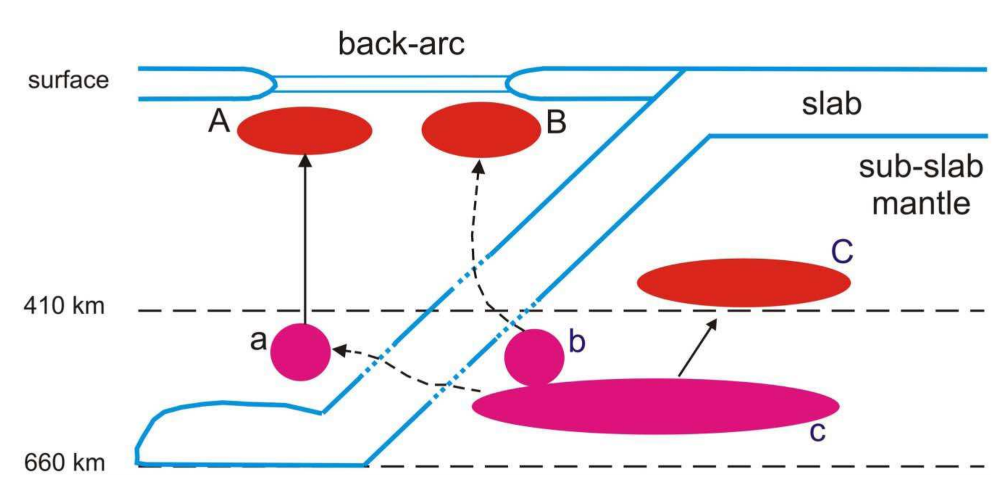
Reconstructing thermal structures in the past. Using the quasireversibility method for data assimilation the set of the discrete equations governing the backward mantle convection flow are solved together with the initial and boundary conditions to restore the mantle evolution in the region of the Japan Islands. In backward sense, the high-temperature patchy anomaly beneath the back-arc Japan Sea basin splits into two prominent anomalies showing two small-scale upwellings beneath the southwestern and northern part of the Japan Sea.



Subsidence history of the Japan Sea: tectonic subsidence curves for the southern Sakhalin section representing the northern margin of theJapan Sea (dotted curve), the Oga Peninsula section representing the inner arc area of north-western Honshu (solid curve), and Dolgorae-1 well representing the southern Tsushima Basin and the southern margin of the Japan Sea (dashed curve). The location of the wells is marked in (b). Three-dimensional view of snapshots of the iso-surfaces of positive (5%) temperature anomalies; colours mark the depth.



The 3D visualization of the temperature anomaly.



Conclusions

It is known that the opening of the Japan Sea has temporal and spatial variations and shows several stages of deformation. Inhomogeneous spreading is consistent with the patchy character of hot materials as evident in our results. Non-instantaneous deformation may imply that the hot materials have penetrated through, or a ected, the overlying subducting Pacic lithosphere several times. The tomography images of a subducting slab usually do not show a single straight high velocity anomaly as one may expect from the forward numerical simulations of subducting lithosphere. Instead they appear to show a couple of high velocity blocks. Such a feature may be explained by an occasional penetration of hot materials below the subducting lithosphere.

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