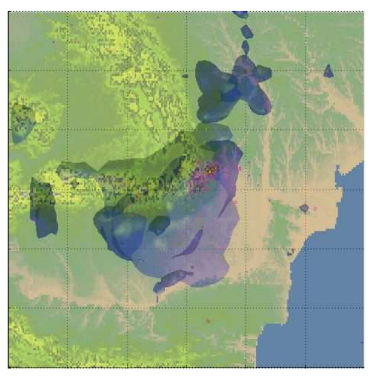

Thermal evolution of the descending lithosphere beneath the SE-Carpathians: An insight from the past

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Mathematical Statement of the Thermal Convection Problem

Governing Equations

Model domain $\Omega = (0, x_1 = l_1) \times (0, x_2 = l_2) \times (0, x_3 = h)$, $t \in (0, \vartheta)$.
Momentum equation

$$-\nabla P + \nabla \cdot (\mu[\nabla \mathbf{u} + \nabla \mathbf{u}^T]) + RaT\mathbf{e_3} = \mathbf{0},$$

Continuity equation $\quad \nabla \cdot \mathbf{u} = 0, \quad x \in \Omega;$
Backward regularized heat equation

$$\frac{\partial T}{\partial t} - \mathbf{u} \cdot \nabla T = -\nabla^2 T - \beta \nabla^4 T$$

State equation $\mu = \exp[Q/(T+G) + Q/(0.5+G)]$, $Q = 225/\ln(r) + 0.25\ln(r)$, $G = 15\ln(r) - 0.5$, $R = 20$

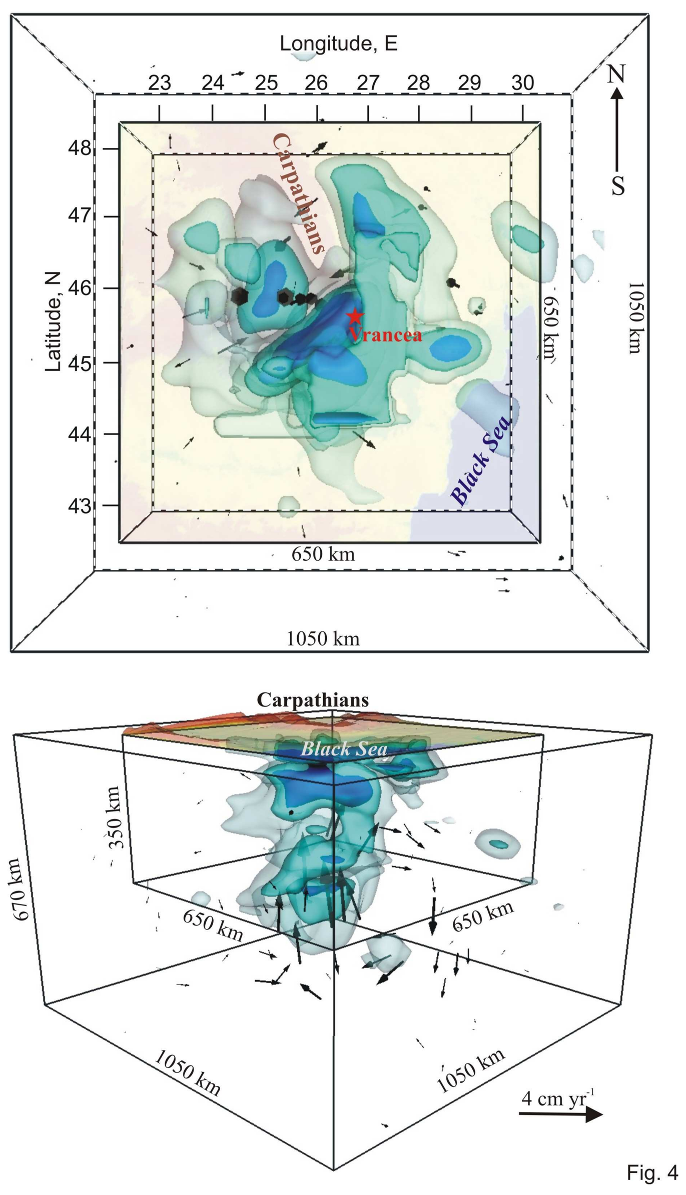
Here $\mathbf{x} = (x_1, x_2, x_3)$, $\mathbf{u} = (u_1, u_2, u_3)$, t, T, P, and μ are the dimensionless Cartesian coordinates, velocity, time, temperature, pressure, and viscosity, respectively; ϑ is the present time; $\mathbf{e} = (0, 0, 1)$; ∇ is the gradient operator; $\nabla\cdot$ is the divergence operator; The Rayleigh number $Ra = \frac{\alpha g \rho_* T_* h^3}{\eta^* \kappa}$, Length, temperature, and time are normalized by h, ΔT, and $h^2\kappa^{-1}$, respectively.

Boundary and Initial Conditions

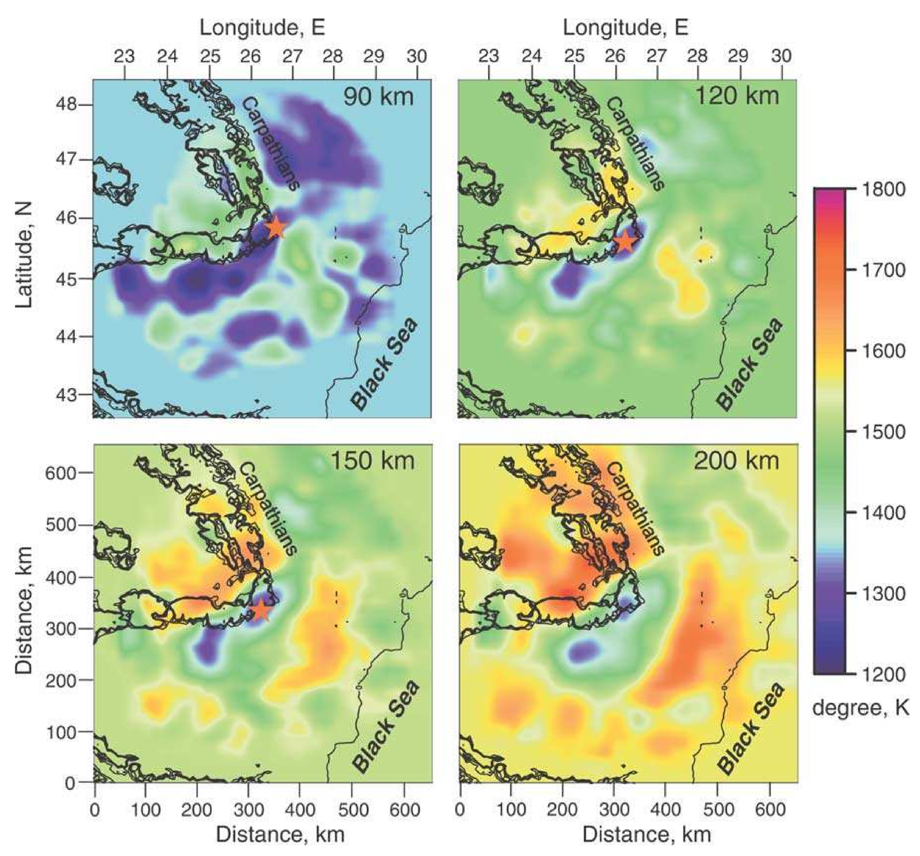
At model boundaries, impenetrability condition with perfect slip

$$\mathbf{u} \cdot \mathbf{n} = 0, \quad \partial \mathbf{u}_\tau / \partial \mathbf{n} = 0, \quad x \in \partial\Omega;$$

Zero heat flux through the vertical boundaries $\frac{\partial T}{\partial \mathbf{n}} = 0$
Isothermal upper and lower boundaries $T = T_{up}$, $x_3 = h$, $T = T_{bm}$, $x_3 = 0$,
Additional boundary condition: $\frac{\partial^2 T}{\partial \mathbf{n}^2} = 0$
At initial time $T(\mathbf{x}, \vartheta) = T^*(\mathbf{x})$.



3-D thermal shape of the Vrancea slab and contemporary flow induced by the descending slab beneath the SE-Carpathians. Upper panel: top view. Lower panel: side view from the SE toward NW. Arrows illustrate the direction and magnitude of the flow. The surfaces marked by blue, dark cyan, and light cyan illustrate the surfaces of 0.07, 0.14, and 0.21 temperature anomaly δT, respectively, where $\delta T = (T_{hav} - T)/T_{hav}$, and T_{hav} is the horizontally averaged temperature. The top surface presents the topography, and the red star marks the location of the intermediate-depth earthquakes.



Temperatures derived from P-wave velocity anomalies beneath the SE-Carpathians at different depths in the mantle. The composition, anharmonicity, anelasticity, and partial melting are taken into account. Isolines present the surface topography. Star shows the location of the Vrancea intermediate-depth earthquakes.

Retrospective Problem of Mantle Convection

- to restore the temperature evolution in the mantle;
- to reconstruct the history of movements of continental plates;
- to find a density distribution of the mantle in the geological past and to compare the observed and modelled amounts and rates of "true polar wander".

Numerical method and Solvers

- To solve numerically Stokes and continuity equations, a two component vector velocity potential is introduces to replace the vector velocity and the pressure in these equations. To compute the velocity potetntial, the Eulerian Finite Element Method with tricubical B-splines basis is emploed (A.Ismail-zadeh at al., 2001)
- Temperature in the heat equation is approximated by Finite Differences and found using the semi-Lagrangian method.
- Numerical codes were designed for parallel performance.
- To solve models of crust/mantle flow, slot $[\vartheta_1, \vartheta_1]$ is divided into m subslot. At each time subslot
  - set of linear algebraic equations is solved by the conjugate
  - gradient method in order to find the vector velocity potential;
  - velocity is determined from the vector potential;
  - the backward regularized heat equation is solved at decreasing regularization parameter b until the difference between two adjacent solutions are smaller than the prescribed value.

Computations

The numerical simulation was performed at supercomputer "Uran" (Institute of Mathematics and Mechanics UB RAS, Yekaterinburg, Russia). For visualisation, we develop a module for the open source visualization application ParaView.

TABLE. Model parameters and values

Parameter	Symbol	Value
Horizontal dimensions	l_1, l_2 ($l_1 = l_2$)	1005 km
Depth of domain	h	670 km
Acceleration due to gravity	g	$9.8ms^{-2}$
Reference temperature	T_{ref}	2000 K
Surface temperature	T_{surf}	300 K
Temperature drop	$T = T_{ref} - T_{surf}$	1700 K
Thermal expansivity	α	$3 \cdot 10^{-5} K^{-1}$
Thermal diffusivity	κ	$10^{-6} m^2 s^{-1}$
Reference density	ρ_{ref}	$3400 kgm^{-3}$
Present time	ϑ	22 Myr
Reference viscosity	μ_{ref}	10^{21} Pa s
Rayleigh number	Ra	$5.2 \cdot 10^5$

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Talks

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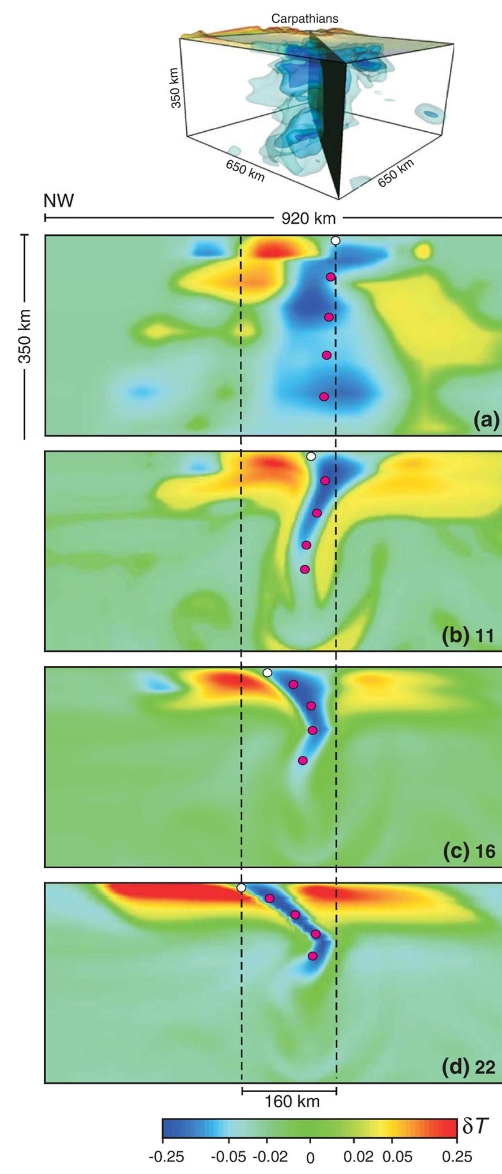
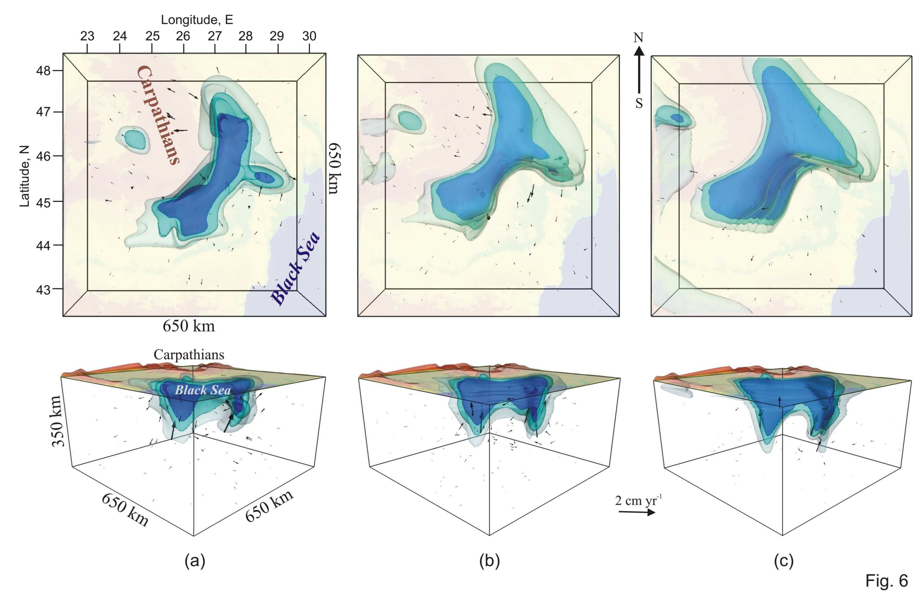
Results of Computation and Visualization

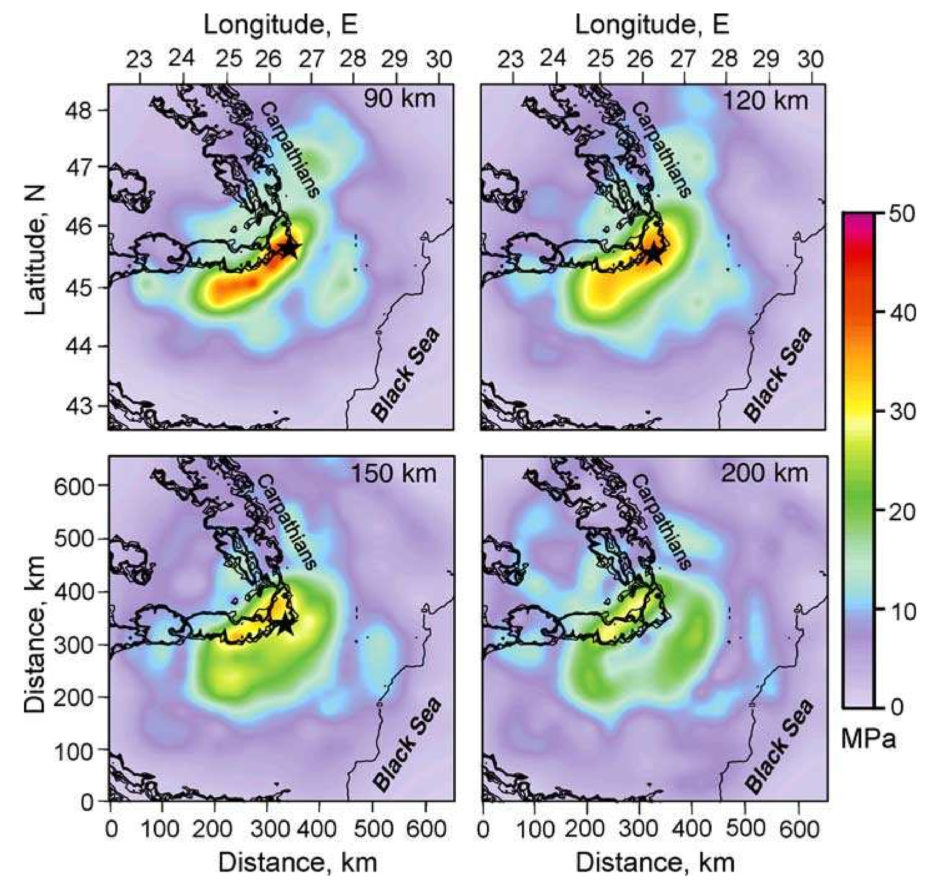
Data assimilation in geodynamical models can be defined as the incorporation of geophysical observations at present and initial physical conditions in the past into a dynamic quantitative model to provide time continuity and coupling among the geophysical fields (e.g., temperature, velocity).



Thermal evolution of the descending Vrancea slab since the Miocene times. Temperature anomalies δT are presented in the NW-SE vertical section (see the upper panel for the section's location). Circles show the location of the passive markers incorporated in the numerical model to display the slab movement. The white circle presents the location of the modeled subduction zone.



Snapshots of the 3-D thermal shape of the Vrancea slab and pattern of mantle flow beneath the SE-Carpathians in the Miocene times.



Maximum shear stress beneath the SE-Carpathians at different mantle depths. Isolines present the surface topography. Star marks the location of the Vrancea intermediate-depth earthquakes.

Conclusions

Using data assimilation we have shown that the geometry of the mantle structures changes with time, diminishing the degree of surface curvature of the structures. Like Ricci flow, which tends to diffuse regions of high curvature into ones of lower curvature (Hamilton, 1982; Perelman, 2002), heat conduction smoothes the complex thermal surfaces ofmantle bodieswith time. Present seismic tomography images of mantle structures do not allow definition of the sharp shapes of these structures. Assimilation of mantle temperature and flow to the geological past instead provides a quantitative tool to restore thermal shapes of prominent structures in the past from their diffusive shapes at present.

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